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Galaxy clustering and the dark-matter problem

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Recent observational and theoretical results on galaxy clustering are reviewed. A major difficulty in relating observations to theory is that the former refer to luminous material whereas the latter is most directly concerned with the gravitationally dominant but invisible dark matter. The simple assumption that the distribution of galaxies generally follows that of the mass appears to conflict with evidence suggesting that galaxies of different kinds are clustered in different ways. If galaxies are indeed biased tracers of the mass, then dynamical estimates of the mean cosmic density, which give $\Omega \approx 0.2$ may underestimate the global value of Ω. There are now several specific models for the behaviour of density fluctuations from very early times to the present epoch. The late phases of this evolution need to be followed by N-body techniques; simulations of scale-free universes and of universes dominated by various types of elementary particles are discussed. In the former case, the models evolve in a self-similar way; the resulting correlations have a steeper slope than that oberved for the galaxy distribution unless the primordial power spectral index $n \approx 2$. Universes dominated by light neutrinos acquire a large coherence length at early times. As a result, an early filamentary phase develops into a present day distribution that is more strongly clustered than observed galaxies and is dominated by a few clumps with masses larger than those of any known object. If the dark matter consists of 'cold' particles such as photinos or axions, then structure builds up from subgalactic scales in a roughly hierarchical way. The observed pattern of galaxy clustering can be reproduced if either $\Omega \approx 0.2$ and the galaxies are distributed as the mass, or if $\Omega = 1$, $H_0 = 50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ and the galaxies form only at high peaks of the smoothed linear density field. The open model, however, is marginally ruled out by the observed small-scale isotropy of the microwave background, whereas the flat one is consistent with such observations. With no further free parameters a flat cold dark-matter universe produces the correct abundance of rich galaxy clusters and of galactic halos; the latter have flat rotation curves with amplitudes spanning the observed range. Preliminary calculations indicate that the properties of voids may be consistent with the data, but the correlations of rich clusters appear to be somewhat weaker than those reported for Abell clusters.

1. Introduction

The clustering pattern of galaxies and the associated velocity correlations are among the most important data for studies of the evolution and present state of the Universe. The mean density of cosmic matter, the nature of the 'missing mass', and the primordial spectrum of density inhomogeneities all influence the growth of structure and are, to some extent, reflected in the observed galaxy distribution. Measured peculiar velocities (that is velocities after subtraction of the Hubble flow) are typically $\lesssim 10^3\ \mathrm{km\ s^{-1}}$. Thus on scales $\gtrsim 10$ Mpc the present-day pattern is likely to retain some memory of its 'initial condition', whereas on smaller scales it

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has been affected by gravitational relaxation processes and perhaps also by radiative and hydrodynamical effects.

There has been a great deal of progress in observational studies of galaxy clustering during the last decade. The early catalogues of galaxy positions on the sky (such as the Zwicky, Shane–Wirtanen or Jagellonian catalogues) have been complemented by redshift surveys either of large areas of sky to fairly bright limiting magnitudes (Davis *et al.* 1982) or of small angular regions to greater depths (Bean *et al.* 1983). These data sample scales in the range *ca.* (0.01–10) h^{-1} Mpc and have been extensively subjected to the statistical analyses developed by Peebles and his co-workers (cf. Peebles 1980). (Here and below the Hubble constant is expressed as $H_0 = 100h$ km s^{-1} Mpc^{-1}.) Scales in the range *ca.* 10–100h^{-1} Mpc have been probed by correlation studies of rich Abell clusters and by rather less systematic searches for filaments and voids. Whereas all these studies have provided some understanding of the three-dimensional distribution of galaxies, they have also raised a major difficulty of interpretation: the observations refer to the distribution of luminous objects but the dynamics of the Universe are determined by the gravitationally dominant but invisible dark matter. What is the connection between them? I will discuss below evidence that suggests that galaxies of different types are clustered in different ways in contradiction with the simple assumption that galaxies are fair tracers of the mass. I shall also argue that there are theoretical reasons to think that bright galaxies may be biased tracers of the mass.

The theoretical framework within which observations of galaxy clustering are normally interpreted is the theory of gravitational instability in an expanding Friedmann universe. The basic ideas stem from the work of Lemaitre (1931) and Lifshitz (1946) and have served as the focus for most of the discussion on this subject. In the past few years a new dimension has been added through input from particle physics, which may provide an explanation for the geometry of the Universe, for the origin of density fluctuations and for the missing mass. In the inflationary model, the early Universe passes through a short-lived phase of exponential expansion at the end of which its curvature is reduced essentially to zero (Guth 1981). Thus this model predicts that the cosmological parameter Ω, defined as the ratio of the cosmic mean density divided by the mean density of an Einstein–de Sitter universe with the same Hubble constant, is very close to 1. Quantum fluctuations in the scalar field whose vacuum energy drives the inflation generate curvature fluctuations; these are causally connected and have a scale-invariant, constant curvature spectrum, known as the Harrison–Zel'dovich spectrum (Harrison 1970; Zel'dovich 1972; Guth & Pi 1982; Hawking 1982; Starobinskii 1982; Bardeen *et al.* 1983). Another important contribution of particle physics to cosmology has been the suggestion that the missing mass may consist of weakly interacting elementary particles. Candidates for the missing mass include light particles such as axions (with rest mass $m \approx 10^{-5}$ eV; Preskill *et al.* (1983)), neutrinos ($m \approx 100$ eV; Gershtein & Zel'dovich (1966); Marx & Szalay (1972); Cowsik & McClelland (1972)), and very massive particles such as photinos, gravitinos or heavy neutrinos ($m \approx 1$ GeV; Cabbibo *et al.* (1981); Bond *et al.* (1982); Olive & Turner (1982); Pagels & Primack (1982); see also Blumenthal & Primack (1983) and references therein). Their properties are reviewed by Ellis (1986, this symposium). Although, with the exception of the neutrino, none of these particles has yet been detected in the laboratory, they are attractive to the cosmologist because they lead to very specific models for the formation of structure that can be tested against observations.

After a brief summary of some relevant observations in §2 I will discuss the linear behaviour

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of various cosmological models in §3 and their nonlinear evolution and comparison with observations in §§4–6. Much of the material in these sections describes work done in collaboration with G. Efstathiou, M. Davis and S. D. M. White. I conclude in §7 with a discussion and summary of results.

2. Observations of galaxy clustering

Observational studies of galaxy clustering reached a peak in the early years of this decade when large surveys of galaxy redshifts became available. Several reviews have been published recently (Oort 1983; Davis 1986) so I will simply summarize here the main results and their interpretation and mention, briefly, two more recent results. A commonly used measure of clustering is the hierarchy of n-point correlation functions of which the lowest order or two-point function has proved particularly useful (cf. Peebles 1980; Fall 1979). For a spatially homogenous sample of mean density n, the two-point correlation function $\xi(r)$ is defined in terms of the probability δp of finding an object in the elemental volume δv at distance r from a randomly chosen object,

$$\delta p = n[1+\xi(r)]\,\delta v. \tag{1}$$

The two-point function for the galaxy distribution has been estimated from catalogues of angular positions on the sky (Totsuji & Kihara 1969; Peebles 1974; Peebles & Hauser 1974) and more recently from redshift surveys (Davis & Peebles 1983; Bean *et al.* 1983; Shanks *et al.* 1983); over the range $r \approx (0.01\text{–}10)\ h^{-1}$ Mpc these estimates are roughly consistent with one another and are well fit by a power law,

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-1.8}. \tag{2}$$

The length r_0 at which $\xi = 1$ is usually referred to as the clustering length and, for the samples mentioned above, $r_0 \approx 5h^{-1}$ Mpc. These estimates apply to optically selected galaxies. Recently Rowan-Robinson & Needham (1985) obtained the projected two-point function of infrared bright galaxies in the IRAS catalogue. Estimating the depth of their sample from the luminosity function of a subset with measured redshifts (Lawrence *et al.* 1986) they found the power-law form of (2) for ξ but with $r_0 = 2.57 \pm 0.18h^{-1}$ Mpc. Because IRAS galaxies are predominantly spirals this result suggests that galaxies of different types may be clustered in different ways. A weaker clustering strength for spirals compared with galaxies as a whole had already been noted by Davis & Geller (1976) from a study of angular correlations in the Zwicky catalogue. Another indication that the galaxy two-point correlation function is not universal comes from the recent analysis by Davis & Djorgovski (1985) of the clustering properties of galaxies as a function of surface brightness. They found $r_0 \approx 5h^{-1}$ Mpc for galaxies with high surface brightness but $r_0 \approx 1.2h^{-1}$ Mpc for galaxies with low surface brightness. If these results are confirmed by further investigations, their cosmological significance is considerable: they imply that galaxies in general are not fair tracers of the mass at least on scales smaller than *ca.* $5h^{-1}$ Mpc and that the process of galaxy formation is sensitive to very large scale effects.

In addition to position correlations, redshift data allow the study of velocity correlations; these two statistics can be related to one another and to the mean density of the Universe through the cosmic virial theorem (Peebles 1976). The physical basis of this relation may be understood by noting that the density contrast of a bound clump of size $r \ll r_0$ and characteristic internal velocity v is $\delta\rho \approx \bar{\rho}\xi(r)$; the standard virial theorem then implies $v^2 \approx G\bar{\rho}\xi(r)\,r$. (The exact relation involves not only the two-point but also the three-point correlation function.) Davis & Peebles (1983) and Bean *et al.* (1983) find that the RMS relative peculiar velocity of galaxy pairs is *ca.* (250–300) km s^{-1} at projected separation $r_p \approx 1h^{-1}$ Mpc; it increases as $r_p^{0.13}$ in the former study and is constant in the latter, in the range $r_p \approx 0.01$–$3h^{-1}$ Mpc. From this they derive $\Omega_g \approx 0.1$–0.2; it is important to note that this estimate includes only that fraction of the mass which is clustered in the same way as the galaxies considered. It is not clear how this value relates to the global parameter Ω; in particular $\Omega = 1$ cannot be excluded provided that most of the mass is more smoothly distributed than the galaxies.

On scales larger than *ca.* $10h^{-1}$ Mpc, the results of galaxy correlation studies are uncertain. There have been various claims of anticorrelations on large scales, but the separation at which ξ is found to be negative varies from one sample to another (Davis & Peebles 1983; Shanks *et al.* 1983). The statistical significance of these results is poor because sampling errors and uncertainties in the background density are comparable to the strength of the correlations on these scales. Possible anticlustering in the galaxy distribution is an important diagnostic; for example in the cold dark-matter model (discussed in §6) $\xi(r) < 0$ is expected for $r > 18h^{-2}$ Mpc (Bond & Efstathiou 1984). The best way to probe scales larger than *ca.* $10h^{-1}$ Mpc to date has been through the distribution of rich Abell clusters. Following on earlier work by Hauser & Peebles (1973), Bahcall & Soneira (1983) and Klypin & Kopylov (1983) found that the two-point correlation function of these objects has the power-law form of (2), but with $r_0 \approx 25h^{-1}$ Mpc. Rich clusters therefore appear more strongly clustered than other forms of luminous matter. Davis (1986) has argued that rich clusters must also be more strongly clustered than the underlying mass distribution; otherwise, the RMS peculiar velocities induced by mass fluctuations would greatly exceed the peculiar velocity of our galaxy relative to the rest frame of the microwave background for any acceptable values of the mean mass density. The enhanced clustering strength of rich clusters has been interpreted by Kaiser (1984) as a statistical bias characteristic of rare events in Gaussian noise fields; high sigma peaks in such fields amplify large-scale correlations preserving their sign (also cf. Barnes *et al.* 1985). Because Abell clusters show positive correlations at $r \approx 50h^{-1}$ Mpc, this effect may also imply positive correlations in the distributions of dark and luminous matter on these scales.

In recent years some attention has been focused on large-scale properties of the galaxy distribution that are not described by the two- or three-point correlation functions, in particular on the existence of filaments and voids. Long chains of galaxies and large underdense regions are visible in projections such as the maps constructed from the Lick catalogue (Kuhn & Uson 1982; Moody *et al.* 1983; Maddox *et al.* 1986); in three dimensions the CfA redshift survey shows some filamentary structure and empty regions (Davis *et al.* 1982; Zel'dovich *et al.* 1982; Huchra *et al.* 1983; cf. figure 6 of Frenk *et al.* 1983; Barrow *et al.* 1985; Soltan 1985), and redshift surveys of superclusters have revealed extensive bridges of galaxies connecting neighbouring clusters (Gregory & Thompson 1978; Chincarini *et al.* 1983; Oort 1983 and references therein; Einasto *et al.* 1984; Batuski & Burns 1985). Binggeli (1982) found a tendency of elongated Abell clusters to point preferentially in the direction of their nearest neighbour. The largest void discovered so far, near the constellation of Boötes, is a region of radius *ca.* $30h^{-1}$ Mpc that is claimed to contain less than 25% of the mean density of bright galaxies (Kirshner *et al.* 1981, 1983), corresponding to a 3σ downward fluctuation (Davis 1985). Most of the evidence for filaments and voids is still somewhat anecdotal, but systematic surveys of such structures, which would be extremely valuable, may come in the not too distant future.

3. The origin of structure in the Universe

In the gravitational instability theory small density irregularities in a uniform expanding background are assumed to have been generated at an early epoch. In the absence of known physical processes that could have introduced phase correlations or preferred scales, primordial density fluctuations are assumed to have random phases and a power spectrum given by

$$|\delta_k|^2_{\mathrm{p}} \propto k^n, \tag{3}$$

where δ_k is the Fourier transform of the fluctuating density field $\delta(\boldsymbol{r}) = \rho(\boldsymbol{r})/\bar{\rho}$ and k denotes spatial frequency. (However, cosmic strings, discussed by Kibble (Kibble & Turok 1986, this symposium) are a possible mechanism to induce strong phase correlations; see also Kibble 1976). Inflation predicts $n = 1$, corresponding to the Harrison–Zel'dovich 'constant curvature' spectrum. As the Universe expands, increasingly long wavelengths come within the horizon and can be affected by causal damping processes that cause different wavelengths to grow at different rates. Thus at time t, still during the linear régime ($\delta \ll 1$), the power spectrum is given by,

$$|\delta_k(t)|^2 = T(k,t)\,|\delta_k|^2_p, \tag{4}$$

where $T(k,t)$ is a transfer function whose form is determined by interactions between the various constituents of the Universe. After (re)combination, the radiation field propagates freely and, unless affected by subsequent re-ionization, any fluctuations present will be reflected today in the microwave background. The high degree of isotropy observed in this background indicates that residual fluctuations at (re)combination were small. Eventually self-gravity leads to the collapse of density inhomogeneities and their subsequent evolution is governed by complex nonlinear processes which ultimately produce the galaxies, clusters and other structures characteristic of our Universe today.

Before the idea that the dark matter might be composed of weakly interacting elementary particles, most discussions on the formation of structure dealt with the evolution of adiabatic and isothermal density fluctuations (see, for example, Silk 1968; Zel'dovich 1967, 1978; Peebles 1974, 1980). Adiabatic fluctuations are perturbations in the space–time metric that involve no variation in the entropy per baryon. This is the mode predicted by most inflationary models but in a purely baryonic universe it leads to temperature anisotropies in the microwave background that exceed current upper limits (Wilson & Silk 1981; Uson & Wilkinson 1984); I will not discuss this case any further. Isothermal fluctuations are perturbations in the entropy per baryon that do not affect the local curvature of space–time; if their primordial spectrum has the power-law form of (3) then, the post (re)combination spectrum is also scale-free over most scales of interest. Although at present there is no physical motivation for a scale-free spectrum, its subsequent evolution is of more than purely academic interest because it can be a good approximation to more realistic models over a wide range of scales. Another advantage of scale-free fluctuations in an Einstein–de Sitter background universe is that their evolution

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may admit a similarity solution from the BBGKY equations (Davis & Peebles 1977). In the next section I will discuss nonlinear evolution from power-law initial conditions with $n = -2, -1, 0$, and 1 in the notation of (3). The two extreme cases correspond to the two straight lines in figure 1 which shows the product $k^3|\delta_k|^2$, a quantity closely related to the RMS density fluctuation on scale $\lambda = 2\pi/k$, plotted against the spatial frequency k.

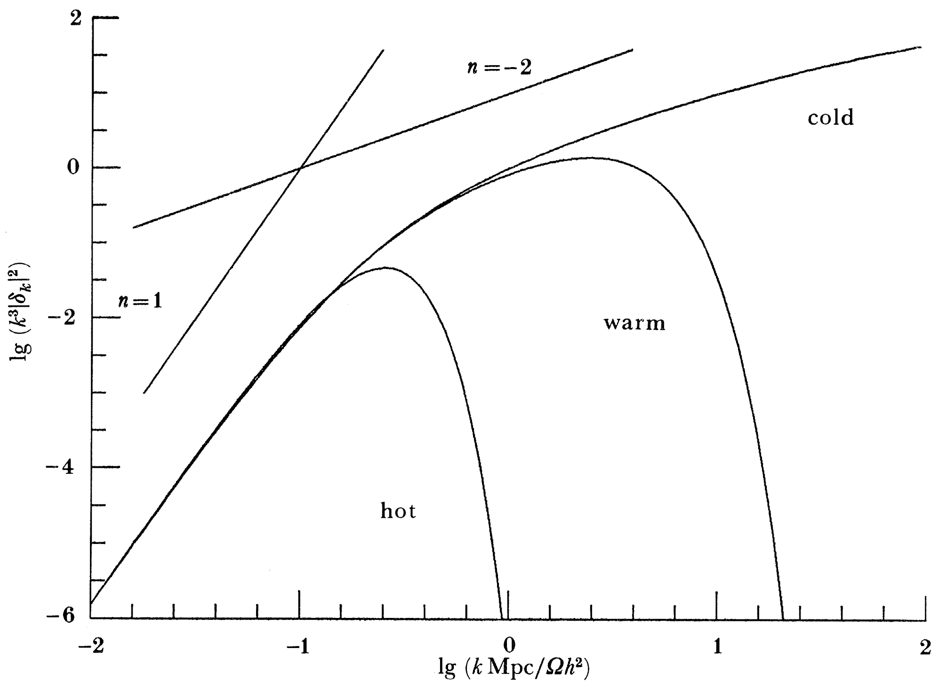

FIGURE 1. Power per decade as a function of spatial frequency for different types of density fluctuations after (re)combination. The two straight lines correspond to scale-free universes with spectral power indices $n = 1$ and $n = -2$ respectively. The three curves correspond to universes dominated by hot, warm and cold dark matter. Adiabatic primordial fluctuations with the Harrison–Zel'dovich spectrum are assumed in these three cases and the curves are taken from the results of Bond & Szalay (1983) and Bond & Efstathiou (1984).

If the Universe is dominated by weakly interacting elementary particles, the transfer function of (4) is determined by two different mechanisms. The first is Landau damping due to free streaming of relativistic particles out of density enhancements; it produces an exponential cut-off in the power spectrum shortwards of a critical wavelength, $\lambda_c \propto m_x^{-1} \propto (\Omega h^2)^{-1}$, where m_x is the particle mass (Bond *et al.* 1980). (The second proportionality follows from the assumption that the Universe is dominated by the x-particles: $\Omega H^2 = \frac{8}{3}\pi G n_x m_x$, where n_x is the number density.) The second mechanism is a reduced growth in the amplitude of matter fluctuations during the radiation era when the dominant photon–baryon fluid undergoes acoustic oscillations; it is known as the Mészáros effect (Guyot & Zel'dovich 1970; Mészáros 1974) and produces a bend in the spectrum from the initial power-law index n to $n-4$, at a characteristic scale $\lambda_c \propto (\Omega h^2)^{-1}$ corresponding to the horizon size at the transition between matter and radiation dominance. Because the importance of free streaming depends on the thermal velocity of the dark matter relative to the Hubble velocity at early times, elementary particle candidates have been classified into three broad classes, known as hot, warm and cold dark matter (Bond *et al.* 1983; see also the review by Primack 1984). The post-recombination fluctuation spectrum of hot dark matter is shaped by free streaming, that of cold dark matter by the Mészáros effect and that of warm dark matter by both effects. These three cases are shown in figure 1 where I have assumed that primordial fluctuations are generated during inflation so they are adiabatic and have the Harrison–Zel'dovich spectrum. The classic example of hot particles are neutrinos with mass of 30 eV for which the cutoff wavelength λ_c corresponds to 41 Mpc today (Peebles 1982; Bond & Szalay 1983). Cold dark matter includes particles which have low velocities at early times either because they are very massive (e.g. photinos, gravitinos or neutrinos with $m_x \approx 1$ GeV) or because they are created in this state (the axion with $m_x \approx 10^{-5}$ eV); other candidates for cold dark matter are quark nuggets (Witten 1984) and primordial black holes (cf. Carr 1978). For the assumed primordial power-law spectrum of index $n = 1$, the cold dark matter spectrum tends asymptotically to a power-law of index $n = -3$ at short wavelengths (Peebles 1982, 1984; Blumenthal & Primack 1983; Bond & Efstathiou 1984). Gravitinos and photinos have been suggested as candidates for warm dark matter (Pagels & Primack 1982; Bond *et al.* 1982) but currently it seems more likely that supersymmetric particles have a mass in excess of 1 GeV; for a hypothetical warm particle with mass *ca.* 1 keV the free streaming cutoff would occur at about a galactic scale.

It is clear from figure 1 that the formation of structure will proceed very differently in universes dominated by hot, warm or cold dark matter. In a neutrino-dominated universe, scales a few tens of megaparsecs across, corresponding to the peak of the power spectrum, are the first to become nonlinear and collapse; clusters and galaxies must subsequently fragment, probably from dense gaseous regions inside pancake-like superclusters. This kind of evolution produces a universe with considerable large-scale coherence and is similar to that originally envisaged by Zel'dovich (1970) for a baryonic model with adiabatic perturbations. In a cold dark-matter universe the fluctuation spectrum has power on all scales; subgalactic size objects are the first to collapse and larger structures must form by gravitational aggregation in a roughly hierarchical way. A similar kind of evolution on scales larger than individual galaxies is expected in a universe dominated by warm dark matter. In all these cases, up to a normalization constant and uncertainties in the cosmological parameters Ω and h, the 'initial conditions' for the growth of structure are completely specified by the power spectra of figure 1 and the assumption of random phases. The qualitative nonlinear features that I have just sketched can actually be calculated in considerable detail, at least in so far as gravity is involved. Thus, unlike many other ideas in cosmology, the hypothesis that the missing mass is composed of elementary particles can be directly tested against observations of the large-scale structure; herein lies one of the attractions of this hypothesis.

In the remainder of this paper I will discuss the later phases of evolution from different types of initial conditions. I will assume that the dynamics of the mass distribution on large scales are governed purely by gravity; this should be a good approximation, particularly if the dark matter consists of collisionless elementary particles. The distribution of luminous material, on the other hand, may be affected also by gas dynamical and radiative processes, even on scales larger than those of individual galaxies where such effects have clearly played a role. However, because the luminous material is dynamically unimportant it is useful to postpone discussion of these effects until the properties of the gravitationally dominant component have been elucidated. The most successful approach to the nonlinear gravitational growth of density perturbations has been the use of N-body simulations. In these, the trajectories of a large number of particles which can be thought of as a representative sample of phase-space elements, are followed by solving the equations of motion in an expanding background universe; the initial positions and velocities of the particles are determined by the linear power spectrum of fluctuations (such as those in figure 1), by an assumption about their phases (usually that they are random) and by the requirement that only growing modes be present. The simulations that I will describe below have been carried out over the past few years in collaboration with G. Efstathiou, M. Davis and S. D. M. White; a detailed account of their technical aspects and a discussion of their range of applicability are given by Efstathiou *et al.* (1985) and results before 1984 have been summarized by White (1985). In most cases we solve for the motion of 32768 particles using the particle–particle/particle–mesh (P^3M) technique whereby forces on large scales are calculated by Fourier methods (with a 64^3 grid) and on small scales by direct summation over neighbouring particles (Efstathiou & Eastwood 1981; Hockney & Eastwood 1981). These parameters provide a dynamic range of about 4000 in mass throughout the calculation.

4. Scale-free universes

Because gravity physics in an Einstein–de Sitter universe involves no preferred mass, time or length scale, the self-similar behaviour of a universe with scale-free initial conditions and its possible relation to the observed Universe have attracted considerable attention (Peebles 1974, 1980; Press & Schechter 1974; Davis & Peebles 1977; Aarseth *et al.* 1979; Efstathiou *et al.* 1979; Efstathiou & Eastwood 1981; White & Negroponte 1982). Although at present there is no physical motivation for such initial conditions, these models are useful for the understanding of the physics of gravitational clustering and may be a good approximation to physical models over a restricted range of scales; in addition they provide a test of the validity of N-body simulations. I will briefly describe here some recent results from an on-going study of scale-free universes, a detailed account of which will be published elsewhere (Frenk *et al.* 1986). In this study we have simulated the evolution of Einstein–de Sitter model universes perturbed by density fluctuations which initially have random phases and power-law spectra with indices $n = -2, -1, 0$ and 1 (equation 3) over the accessible range of scales. To set up initial conditions we place $N = 32768$ particles on a cubic grid and perturb their positions and velocities using Zel'dovich's formulation of linear theory, as described by Efstathiou *et al.* (1985). On scales smaller than the side L of the computational volume and larger than the Nyquist frequency of the particle grid, $\pi N^{\frac{1}{3}}/L$, the particle distribution corresponds to a random realization of the desired power-law spectrum.

Figure 2 shows three epochs in the evolution of the models. These diagrams are two-dimensional projections of the three-dimensional particle distributions plotted in comoving coordinates; time runs horizontally and each row corresponds to a different model with n ranging from 1 at the top to -2 at the bottom. The expansion factors were chosen so that the mass that is just becoming nonlinear at each output time is the same multiple of the initial nonlinear mass in all cases. Thus, at each of the epochs shown, the clustering has grown by a similar factor in all the models. The left hand panels represent an early epoch in the simulations; as n decreases the relative power on large scales increases and as a result small-scale perturbations in the $n = -2$ model are already nonlinear whereas in the subrandom $n = 1$ model the distribution is nearly uniform. Structure evolves very rapidly in the $n = -2$ model and progressively more slowly for larger n. The $n = -2$ model develops a wide range of structures from small groups to loose superclusters of clumps and has a vague filamentary

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appearance. By contrast, the $n = 1$ model is characterized by a large number of nearly homogeneously distributed small tight clumps. The $n = 0$ and $n = -1$ models exhibit an intermediate behaviour between these two extremes. In all the models substructure appears to be erased as clumps merge into larger and larger systems. By the final time shown the distribution is dominated by a few large clumps in the $n = -2$ model and by a larger number of smaller and tighter clumps in the $n = 1$ model; the central regions of these clumps appear to be rather smooth. Eventually the clustering scale approaches the box size as the fundamental mode in the calculation becomes nonlinear; the models are then unrealistic and the integrations must be stopped. This occurs after only an expansion factor of about 3 in the $n = -2$ model but not until an expansion factor of nearly 70 in the $n = 1$ model. Because of the narrow time window available in the $n = -2$ model, small-scale transients caused by numerical inaccuracies in the initial conditions, do not completely die out before the end of the calculation. This prevents this simulation from fully reaching the self-similar régime. Such effects, however, become negligible for larger values of n.

The formation histories of two representative clumps in an $n = 1$ and an $n = -2$ simulation are depicted in figure 3. Here I have plotted the projected positions, at three epochs, of the particles that end up in the third most massive clump identified in each model at the final time; physical (rather than comoving) coordinates are used so the perturbations can be seen to expand to a maximum radius and then collapse. The evolution of the two clumps is rather different. In the $n = 1$ model the final clump originated from a large number of similar tight subclumps which coalesced on a slow timescale. In the $n = -2$ model it originated from a large condensation which is already fairly smooth at early times and rapidly accretes various smaller loose subcondensations of different masses. In both cases the final distributions are smooth, triaxial and centrally concentrated. This behaviour and that of the simulations as a whole are reflected in the multiplicity function defined as the fraction of the total number of particles which are found in groups of different sizes. In the $n = 1$ model it is sharply peaked whereas in the $n = -2$ model it is a broad function of the group multiplicity (cf. also Bhavsar *et al.* 1981). A detailed discussion of these and other properties of clumps as a function of spectral index will be given in our forthcoming paper.

The self-similar behaviour of the models is clearly demonstrated in the evolution of their two-point correlation functions. These are plotted in figure 4 for all four models and represent ensemble averages over four simulations with different statistical realizations of the same initial power spectra. In an Einstein–de Sitter universe with scale-free fluctuations the correlation function $\xi(x, t)$, where x is comoving distance, and t is time, obeys a similarity transformation,

$$\xi(x,t) = \hat{\xi}(s), \quad s \propto x t^{-4[3(n+3)]}. \tag{5}$$

In an N-body simulation there are two artificial length scales, a comoving softening length η and the size of the computational volume L. Self-similarity is not expected to hold on scales smaller than η or at times when the clustering length approaches L (Efstathiou *et al.* 1985). The scaling implied by the similarity solution is indicated by the solid line in figure 4; this corresponds to $\xi(x, t)$ at the third to last output time shifted horizontally by the amount given in (5) for each output time. If self-similarity is obeyed this curve should match the plotted points at all output times. This is indeed the case for the models with $n > -2$ on all scales larger than η (indicated by an arrow in figure 4) and at times less than the final one shown. For $n < 1$,

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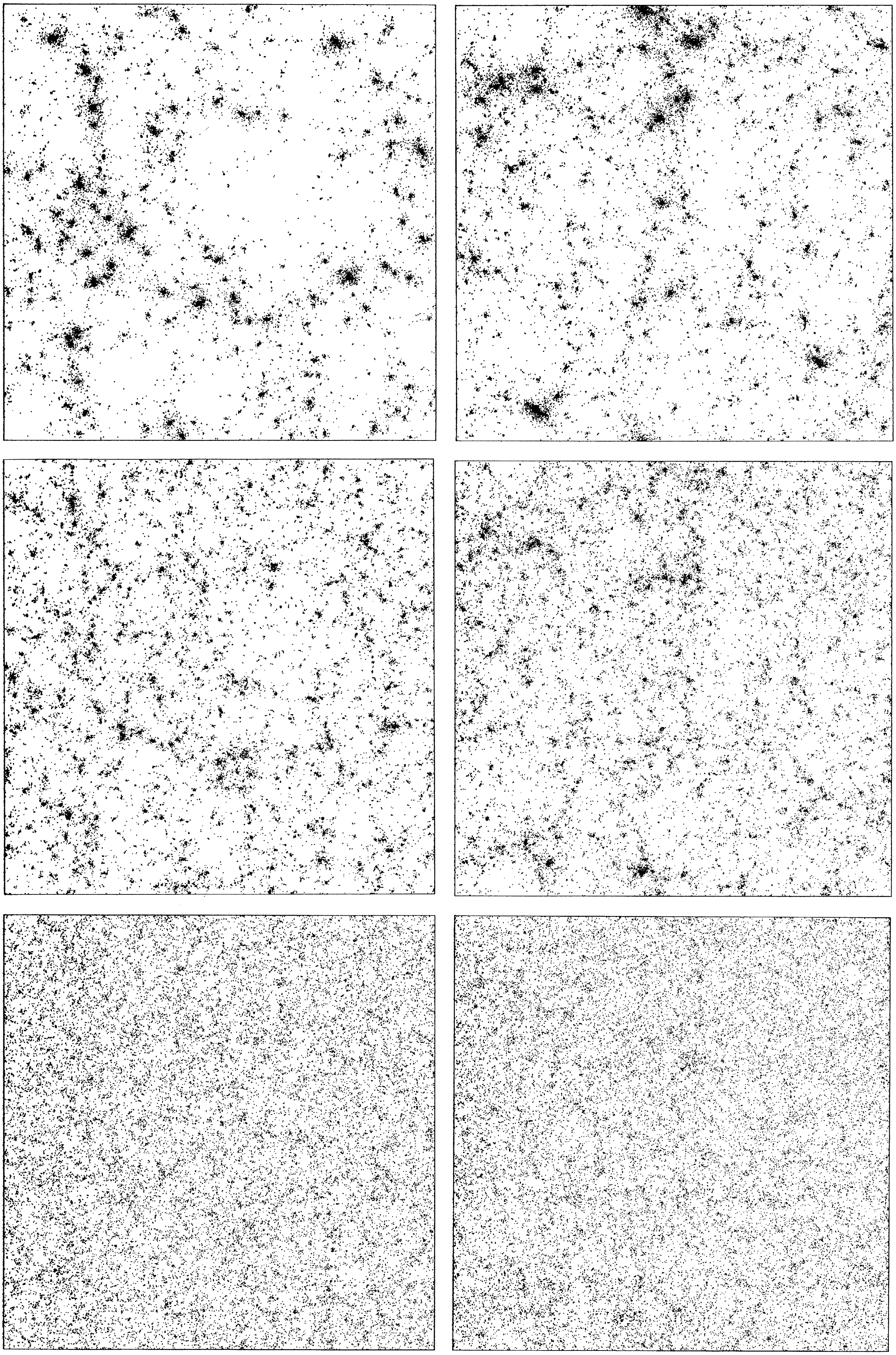

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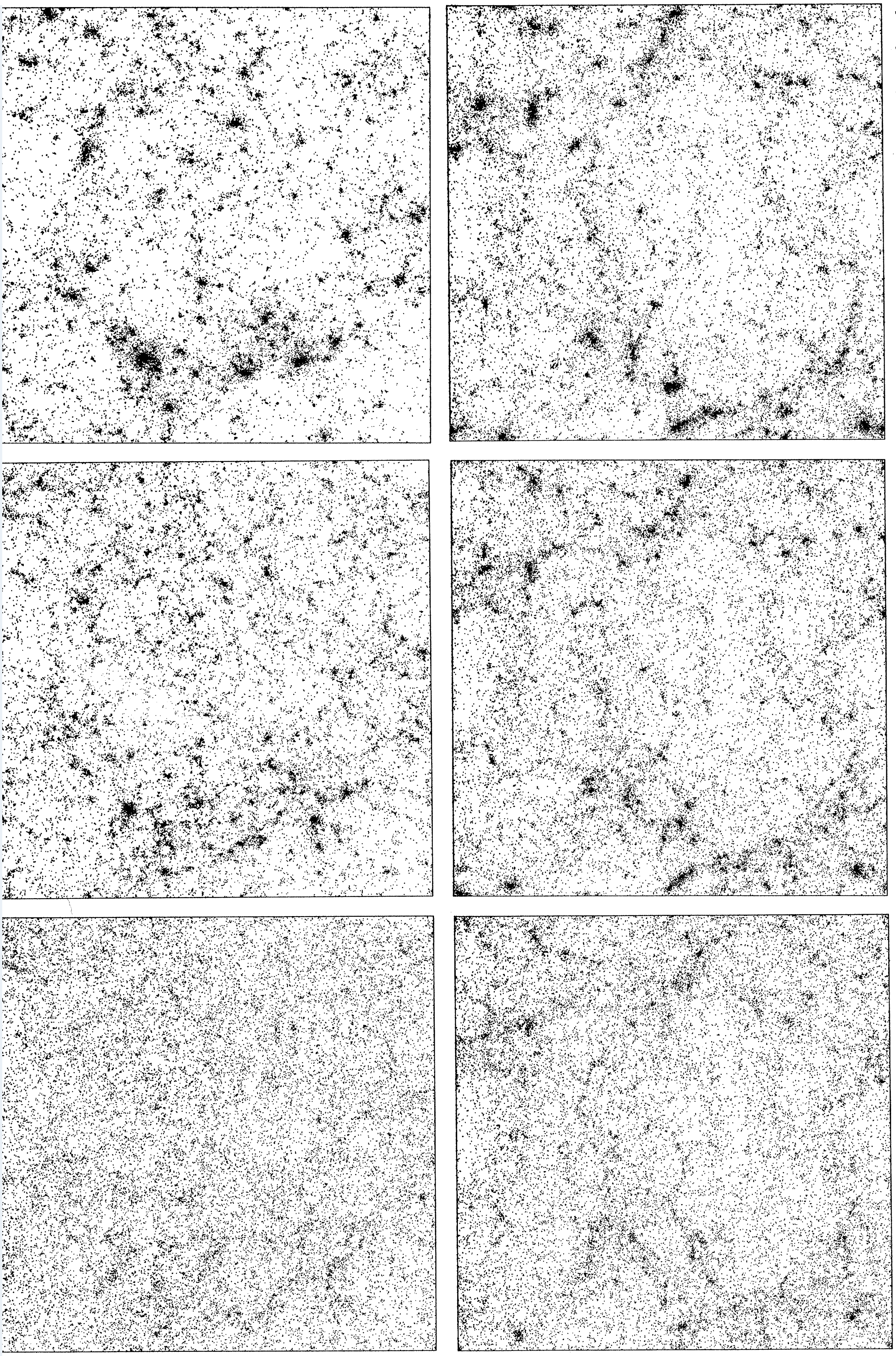

FIGURE 2. Projected positions of particles at three epochs in N-body simulations of scale-free universes. Comoving coordinates are used. Time increases from left to right and each row corresponds to a model of different spectral power index (cf. (3)) with $n = 1, 0, -1$ and -2 from top to bottom. The expansion factors shown are: (top row) 3.3, 20.1, 66.6 ($n = 1$); (second row) 2.5, 9.5, 23.4 ($n = 0$); (third row) 1.8, 4.5, 8.2 ($n = -1$); (bottom row) 1.3, 2.1, 2.9 ($n = -2$).

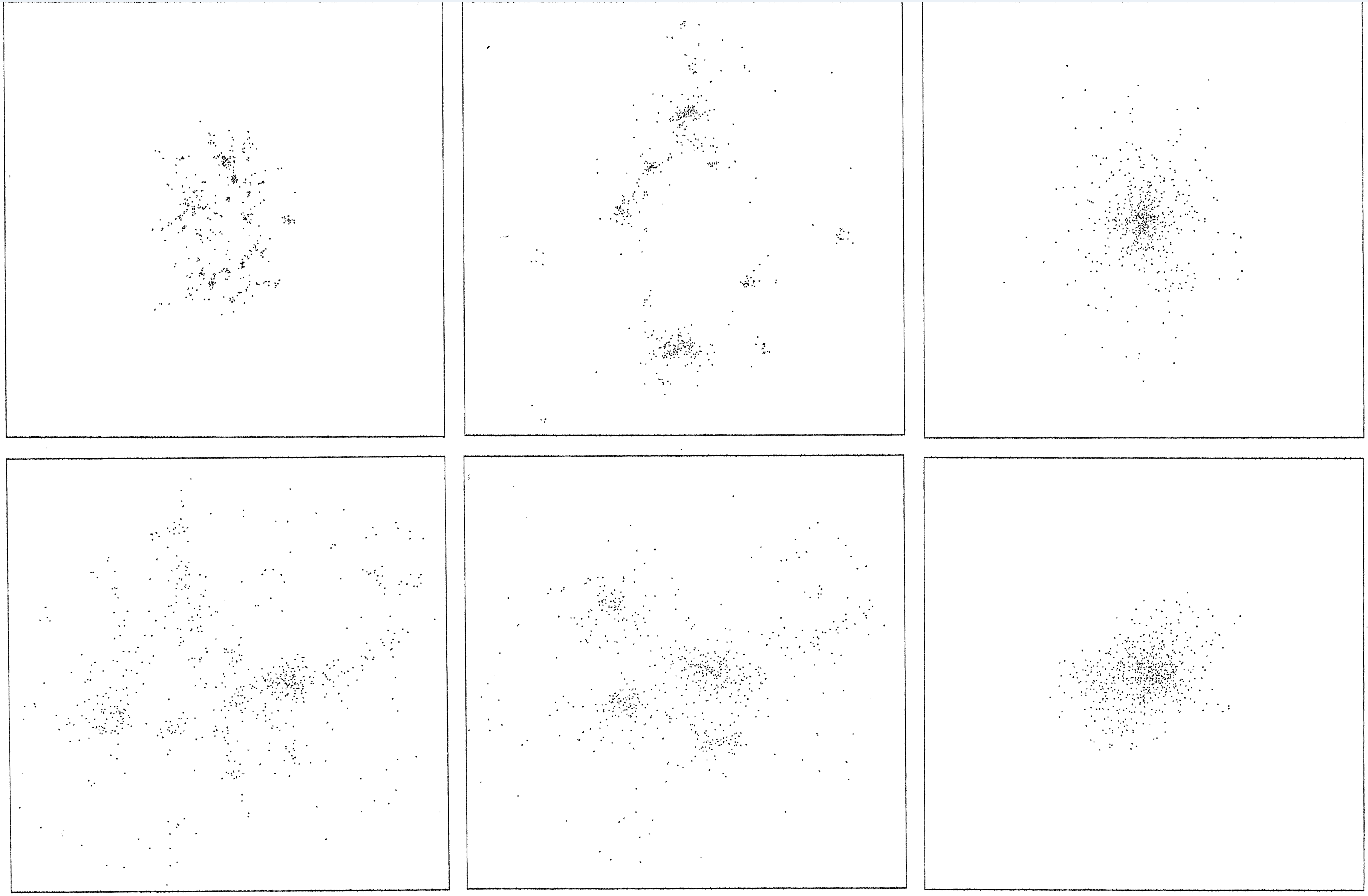

FIGURE 3. Projected positions at three epochs of the particles that end up in the third most massive group in scale-free simulations with $n = 1$ (top) and $n = -2$ (bottom). Physical (not comoving) coordinates are used. Time increases from left to right and the expansion factors shown are 6.1, 20.1, 66.6 ($n = 1$) and 1.6, 2.1, 2.9 ($n = -2$). The groups were identified at the last time shown by using the percolation algorithm described by Davis *et al.* (1985) to link particles separated by less than 0.25 of the mean interparticle spacing; the number of particles and final density contrasts are 601 and *ca.* 2000 ($n = 1$) and 723 and *ca.* 1000 ($n = -2$).

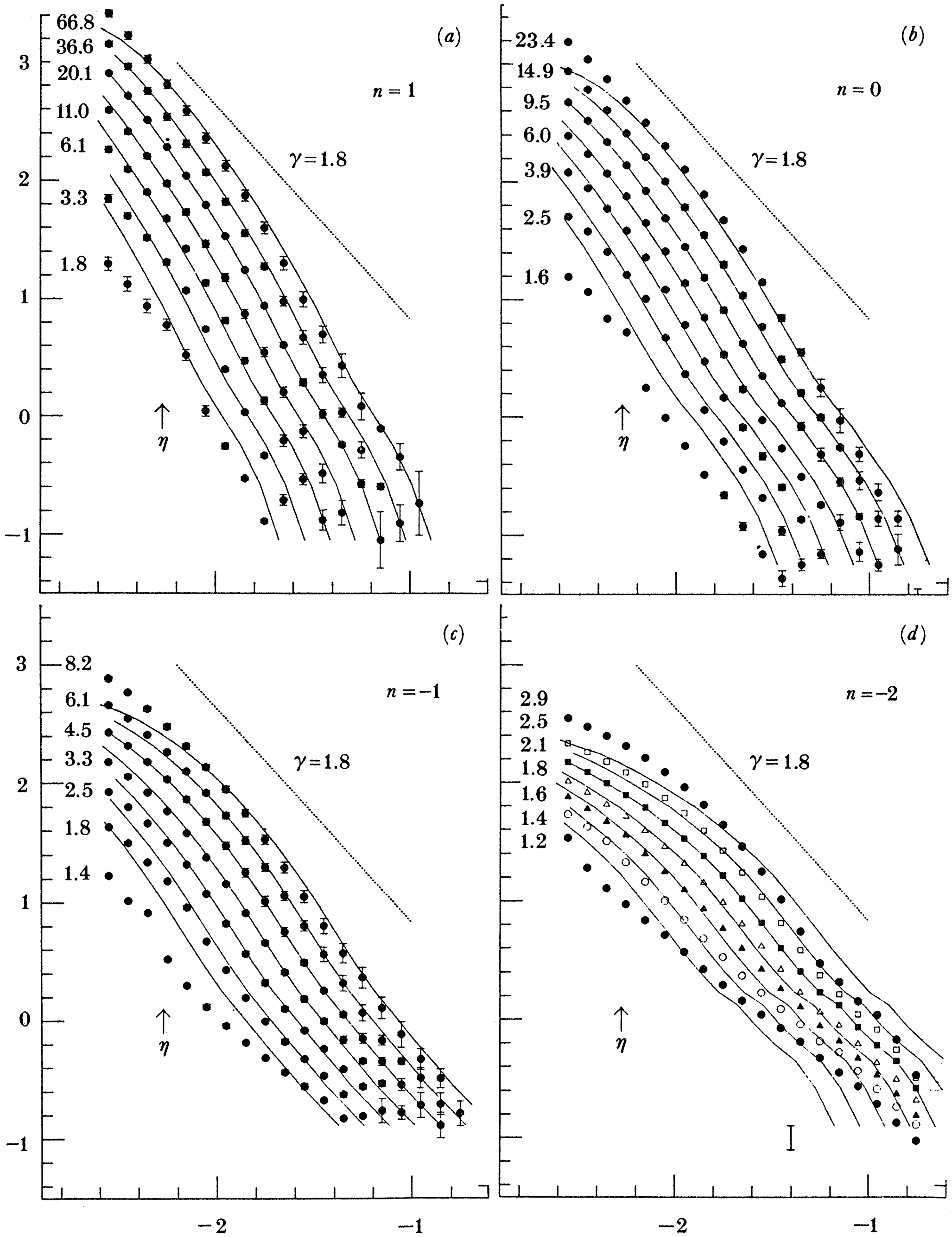

FIGURE 4. Two-point correlation functions for ensembles of simulations of scale-free universes. The spectral power index in each case is shown at the top right-hand corner and the expansion factors are given at the left of each curve. The solid curves represent the correlation functions at the third from last time shown scaled to the remaining times according to the similarity solution (5). The straight dotted line has the slope of the observed galaxy correlation function.

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small deviations on large scales are apparent at this time reflecting the growing importance of boundary effects. The $n = -2$ model obeys the self-similar scaling approximately and only during a short interval of time after the initial transients have subsided but before the clustering length has become too large. The agreement with the similarity solution exhibited by the correlation functions is shared by other properties of the models such as their velocity distributions and multiplicity functions.

Peebles (1974) and Davis & Peebles (1977) showed that if in addition to self-similarity it is assumed that the clustering is in statistical equilibrium then, at small separations, the correlation function must be a power law of index γ related to the spectral index n by $\gamma = (9+3n)/(5+n)$. The N-body simulations, however, show that this behaviour does not occur for $1 < \xi < 10^2$. Over this range, ξ deviates considerably from a power-law form even on scales unaffected by softening. Over the restricted range $1 < \xi < 10$ (where the galaxy correlations are best observed), ξ is approximately fit by a power law with index -2.9 for $n = 1$, -2.6 for $n = 0$, -2.4 for $n = -1$ and -1.9 for $n = -2$. This discrepancy with the analytic predictions arises because the assumption of statistical equilibrium is not satisfied in the N-body simulations; the peculiar velocities of pairs at separations corresponding to $\xi \approx 1$ exceed the Hubble velocity in absolute value indicating infall on these scales (cf. also Efstathiou & Eastwood 1981; Efstathiou *et al.* 1985; Davis *et al.* 1985). This is the régime most accurately modelled by N-body simulations. The results presented here indicate that the observed pattern of galaxy clustering did not arise from random phase, scale-free fluctuations with $n \gtrsim -2$.

5. Neutrino universes

At first sight neutrinos appear to be very attractive candidates for the missing mass. In contrast with all other elementary particle candidates they are known to exist and a measurement of a neutrino mass of about 30 eV was reported a few years ago (Lubimov *et al.* 1980; Reines *et al.* 1980). Such a mass would resolve a number of important cosmological issues: it implies that $\Omega = 1$ and it singles out a preferred length-scale in the early universe which corresponds roughly to the scale of present-day galaxy superclusters. Unfortunately, when it was subjected to closer scrutiny this idea did not live up to such expectations mainly because the predicted large-scale structure does not agree with observations (White *et al.* 1983, 1984). As discussed by Jelley (1986, this symposium) the early reports of a neutrino mass have not been confirmed. The cosmological arguments against massive neutrinos have been summarized by White (1985). Nevertheless, it might be instructive to review briefly the large-scale properties of neutrino universes as a counterpoint to the other models discussed in this article.

The dynamical evolution of a neutrino universe is dominated by the sharp cutoff in the fluctuation spectrum at a present-day scale of a few tens of megaparsecs (see figure 1). This coherence length turns out to be much too large to be consistent with the observed clustering scale of galaxies. The neutrino distribution at late times obtained from the N-body simulations of White *et al.* (1983), is shown in figure 5 for an Einstein–de Sitter background universe. It has a marked filamentary appearance which reflects the coherence length of the initial power spectrum (see also the simulations by Klypin & Shandarin (1983), Frenk *et al.* (1983) and Centrella & Mellot (1983)). However, the filaments which join rich clusters and surround large low-density regions are a transient feature; soon after the epoch shown in the figure they break up into clumps and at a later time the distribution is completely dominated by a few large

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clusters containing most of the mass. In order to decide which of these epochs should be compared to the present, it is necessary to relate the neutrino distribution to the observed galaxy distribution. Galaxies can only form after the collapse of the first nonlinear structures; in the simulations we can identify the onset of galaxy formation with the time when 1% of the initial Lagrangian grid elements, used to specify the initial conditions, have collapsed to zero volume. The particles that have passed through these possible sites of galaxy formation are labelled as 'galaxies' and are shown in figure 5. If the model time corresponding to this figure is identified with the present, galaxy formation would have occurred at a redshift of 1.1. This is far too recent, in conflict with the lower limit of 5 claimed by Cowie (1985) for the redshift of galaxy formation and with observations of quasars at redshifts of *ca.* 3. To avoid this problem we must identify a later, more clustered, epoch in the simulation with the present. A universe in which galaxy formation occurred at a redshift of 2.5 is illustrated in figure 6*b*. To facilitate comparison with observations, this figure represents the model as seen by a hypothetical observer located at random in the computational volume who can only 'see' particles selected with the same selection function of the CfA redshift survey (Davis *et al.* 1982; Davis & Huchra 1982). Both, neutrinos (open circles) and 'galaxies' (small dots), are plotted. The corresponding projection of the CfA survey galaxies is shown in figure 6*c*. It is clear that the level of clustering of the neutrinos and even more so of the 'galaxies' in the simulation far exceeds that of the galaxies in the Universe. The discrepancy is even worse in an open universe. The predicted distributions are in disagreement with the observations for any acceptable redshift of galaxy formation and for any light neutrino mass.

The arguments of the previous paragraph rely to some extent on the identification of galaxies in a simulation of the mass component. Although our identification algorithm is somewhat arbitrary, most conceivable physical mechanisms for galaxy formation would lead to a more clumpy distribution for the galaxies than for the neutrinos and would not therefore alter our

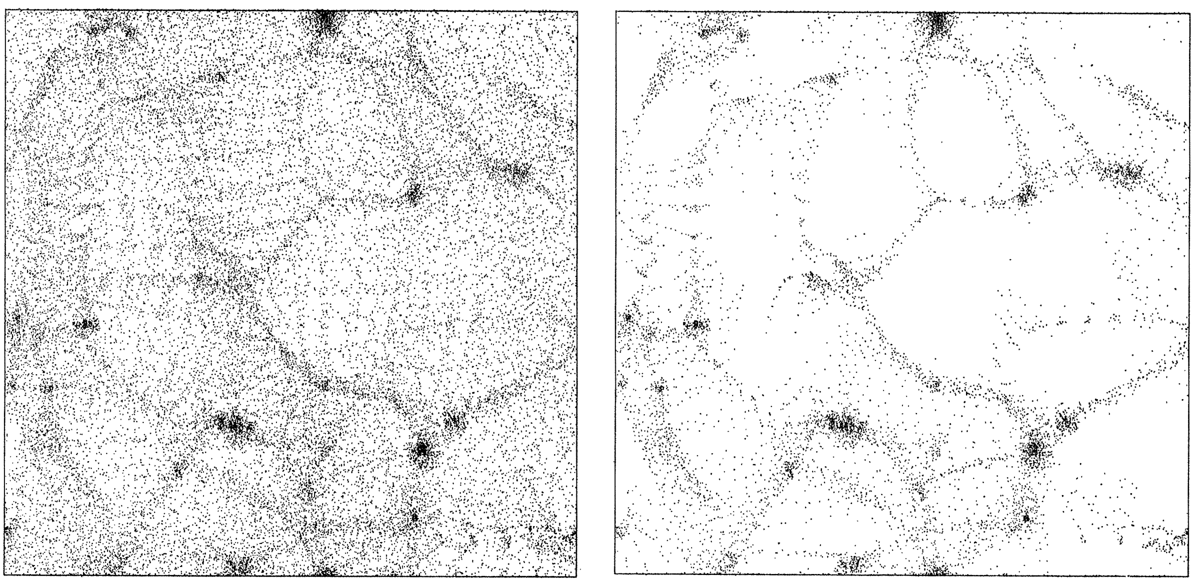

FIGURE 5. Projected particle positions at one epoch in a simulation of a Einstein–de Sitter neutrino-dominated universe. The left-hand panel shows all the particles in the simulation while the right-hand panel shows only those particles identified as 'galaxies'. These distributions correspond to a universe in which galaxy formation began at a redshift less than or equal to 1.1. The side of the box is $65h^{-2}$ Mpc.

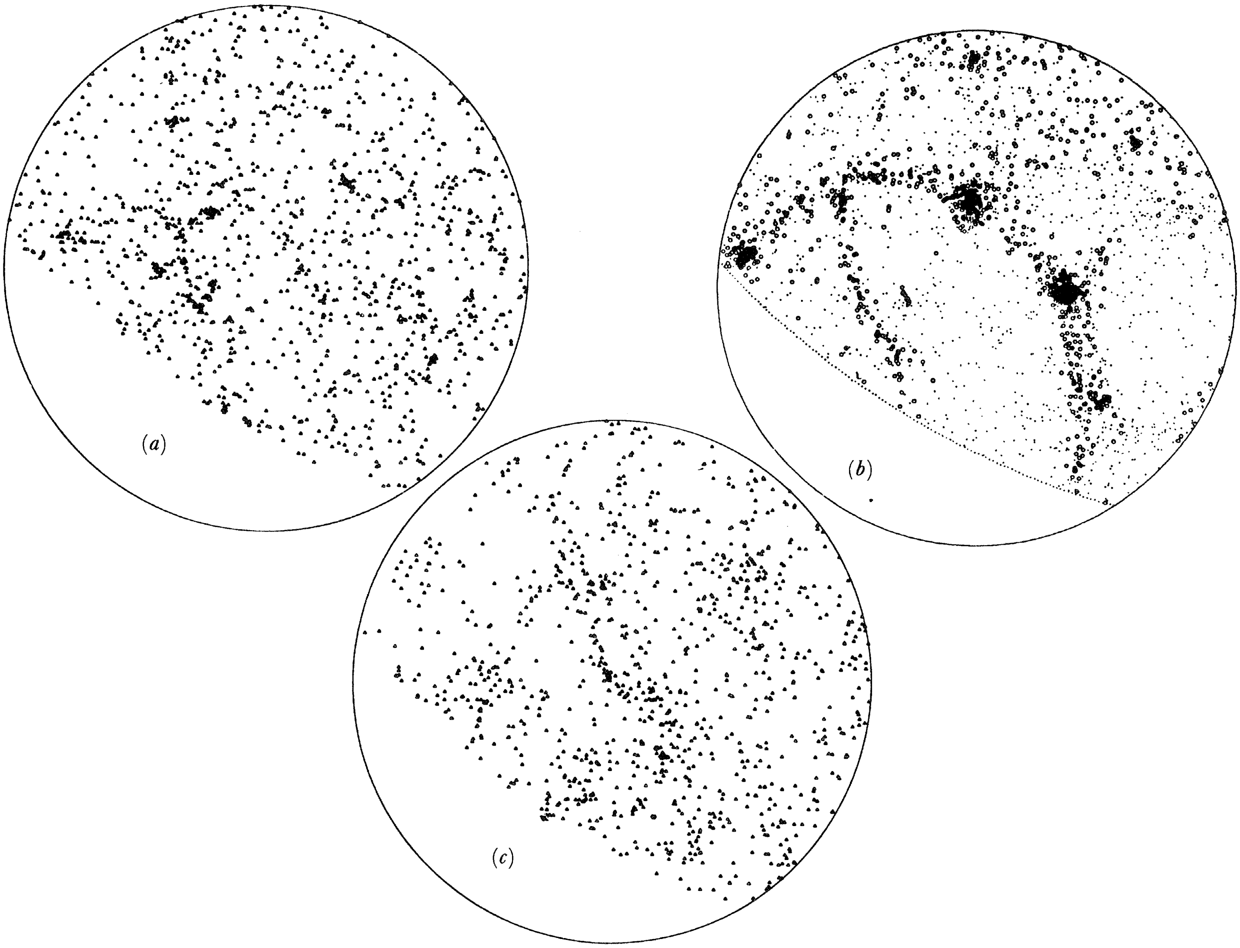

FIGURE 6. Equal-area projections on the sky of the galaxy distribution (*c*) and of particle positions in simulations of (*a*) an open ($\Omega = 0.2$) cold dark-matter universe and (*b*) an Einstein–de Sitter neutrino universe. Particles were selected in such a way as to mimic the CfA northern survey catalogue assuming a hypothetical 'observer' located at random in the simulation. The data in the CfA catalogue are shown in (*c*); the outer circle represents galactic latitude $+40°$, and the empty regions lie at declinations below 0°. The cold dark-matter model is shown at an expansion factor of 3.2 when, for $h = 1.1$, the correlation function of the mass has the same slope and amplitude as that of the observed galaxies. The neutrino model corresponds to a universe in which galaxy formation began at a redshift less than or equal to 2.5; the dots represent the neutrino distribution whereas the open circles represent 'galaxies'.

conclusions. Nevertheless it might be argued that we know so little about galaxy formation that perhaps some way around these problems can be found. To allow for this possibility we considered the properties of neutrino clumps in our simulations, without making reference to the galaxy distribution (White *et al.* 1984). At the present epoch these clumps would contain about half the mass of the universe and their individual masses are so large that they would have no counterparts in the observable universe. Accretion of a small amount of baryons onto these clumps would turn them into strong X-ray sources which are not observed. Another idea that has been put forward to salvage the neutrino model is that some fraction of the neutrinos have a mass greater than 30 eV (and hence give rise to a smaller coherence length) but, in order to preserve a flat universe, they are assumed to have decayed non-radiatively into relativistic particles (Doroshkevich & Khlopov 1983; Hut & White 1984; Turner *et al.* 1984;

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see also Schramm 1985 and references therein). There is not much physical motivation for this idea and, moreover, Efstathiou (1985) has argued that it is not possible to obtain bound galaxy clusters in this model without at the same time causing too large an acceleration of the Milky Way relative to the Virgo cluster of galaxies. Other arguments against massive neutrinos are the large streaming velocities predicted in this model (Kaiser 1983) and the phase-space constraints which prevent neutrinos from condensing into halos of dwarf spheroidals, if those halos do indeed exist (Faber & Lin 1983). Although it may be that a sufficiently contrived model can overcome all these objections, at the present time it appears extremely unlikely that massive neutrinos are the missing mass.

6. Cold dark-matter universes

(a) *Galaxy clustering*

The most successful model of galaxy clustering, so far, assumes that the dark matter is a cold collisionless relic (Blumenthal *et al.* 1984; Davis *et al.* 1985). Although this model is by no means yet firmly established, it has survived at least as much scrutiny as the neutrino model. The standard post-recombination fluctuation spectrum for cold dark matter tends to the primordial power law of index $n = 1$ on large scales and gradually tilts to an asymptotic power law of index $n = -3$ on small scales (see figure 1). Thus, in contrast with the neutrino universe, large-scale structure in this model builds up from subgalactic scales in a roughly hierarchical fashion. Because the spectrum is very flat at high frequencies, a wide range of scales become nonlinear almost simultaneously; *N*-body simulations from these initial conditions require a large dynamic range and a high-resolution code is essential. An important consequence of the 'crosstalk' between different scales is that the process of galaxy formation may be significantly influenced by environmental effects. This makes the connection between the distributions of galaxies and of cold dark matter difficult to model. The uncertainties in this case are greater than in a neutrino universe where a simple prescription for locating galaxies such as the one described in the preceding section seems quite adequate.

The formation of large-scale structure in a cold dark-matter universe was recently studied by a group of us, using the P^3M simulation technique (Davis *et al.* 1985). We found two models which are in good agreement with the galaxy data. The first is an open model with $\Omega = 0.2$ at present, in which the galaxies are assumed to be fair tracers of the mass. The predicted galaxy distribution is displayed in figure 6*a*; its clustering pattern, which has a trace of filamentary structure, is quite similar to the observations (figure 6*c*). This visual agreement is reflected in the two-point correlation functions of the model and the observations: they have the same power-law slope and the same amplitude for $h \approx 1$. (The model correlation function is a power law that steepens slowly with time; equally good agreement with the data but for a lower value of h would have been obtained if we had assumed a lower initial fluctuation amplitude in our models.) The three-point function and the peculiar velocity distribution in the simulations do not match the galaxy data in detail, but the differences are not great. A more worrisome difficulty with this model is that it predicts fluctuations in the microwave background on small angular scales, which marginally exceed the current upper limits if $\Omega h^{\frac{4}{3}} < 0.2$ (Bond & Efstathiou 1984; Vittorio & Silk 1984; Uson & Wilkinson 1984). In addition an open model has the disadvantage that it conflicts with theoretical prejudice and with the standard inflationary model, both of which demand that $\Omega = 1$.

Not surprisingly, an $\Omega = 1$ model in which the galaxies are assumed to trace the mass does not match the galaxy data. The large-scale topology of the flat model is similar to that of the open one and the correlation function also steepens slowly with time. When the slope of this function is 1.8, the distribution of the dark matter is significantly more uniform than that of the CfA galaxies (or equivalently, an unacceptably low value of h, $h = 0.25$, is required to force agreement between the clustering lengths of the model and the observations). Correspondingly, the RMS peculiar velocity of pairs in the model is about 1900 km s^{-1}, much greater than the observed values of about 400–700 km s^{-1}. This discrepancy is not at all unexpected because dynamical estimates of the density of matter clustered as galaxies on scales of a few megaparsecs give $\Omega \approx 0.2$ (see §2). A flat universe can only be reconciled with these estimates if the galaxies are more strongly clumped than the mass. This situation may, in fact, arise quite naturally in a cold dark-matter universe where environmental effects might be strong enough to restrict galaxy formation to regions of unusually high density. Rees (1985) has discussed various physical mechanisms that could bring this about and concludes that ultraviolet radiation pressure coupled to the intergalactic gas via Lyman α scattering may inhibit galaxy formation in low-density regions (see also Silk 1985). In our simulations we assumed that only particles nearest those peaks of the smoothed linear density field that lie above a global threshold would turn into bright galaxies. Two parameters are required to specify this process: the width, s, of the Gaussian filter used to smooth the density field of our initial conditions and the height above threshold which we quantify as ν times the RMS fluctuation. Our prescription selects a biased subset of the particles; they follow the overall clustering pattern of the underlying mass distribution but with greatly enhanced amplitude. This is illustrated in figure 7 which shows

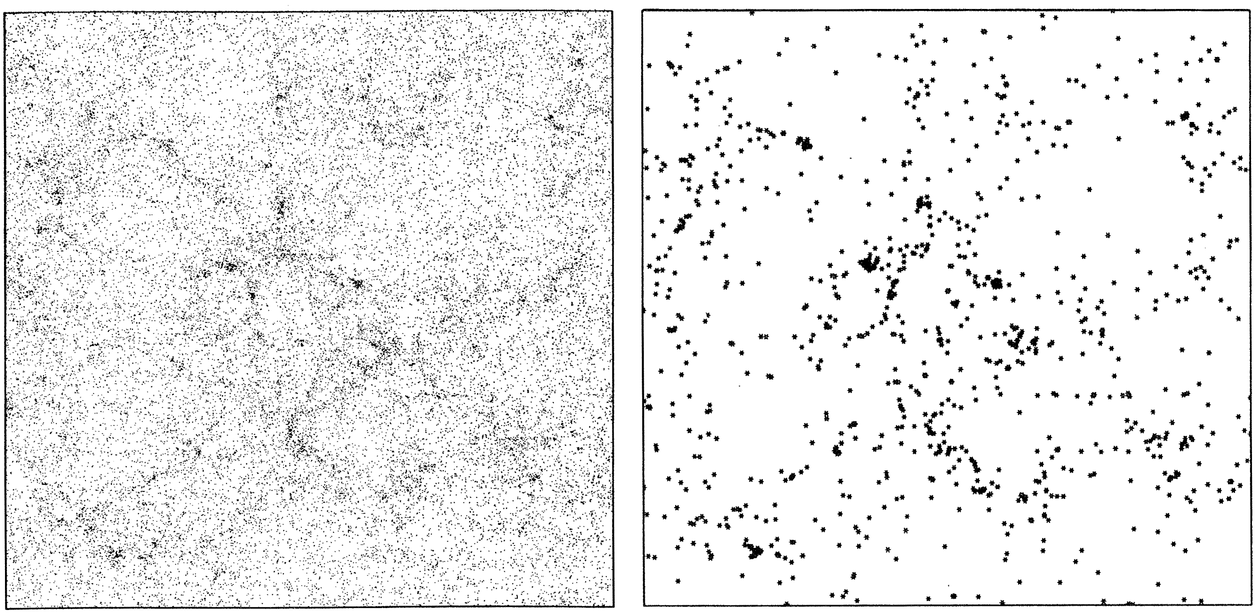

FIGURE 7. Projected particle positions at one epoch in a simulation of an Einstein–de Sitter universe dominated by cold dark matter. The left-hand panel shows all the particles in the simulation while the right-hand panel shows those particles identified as 'galaxies'. These are assumed to form only at the 2.5σ peaks of the linear density field. The expansion factor is 1.4 and at this time the correlation function of the 'galaxies' has a slope and amplitude similar to that of the observed galaxy distribution. The side of the box is $32.5h^{-2}$ Mpc and the value of h required for agreement with observations is $h = 0.44$.

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the distributions of dark matter (left-hand panel) and 'galaxies' with $\nu = 2.5$ and $s = 0.005\,L$, (right-hand panel) in a box of size $L = 32.5(\Omega h^2)^{-1}$. The correlations present in the dark matter are substantially amplified in the 'galaxies' by the same effect invoked by Kaiser (1984) to explain the large correlations of Abell clusters (Kaiser 1986; Bardeen 1986; Schaeffer & Silk 1985; Bardeen *et al.* 1986). The nett result is to bring a flat cold dark matter universe into agreement with observations. Indeed, for $h = 0.5$ the 'galaxies' plotted in figure 7*b* have approximately the same number density as bright galaxies, similar two- and three-point correlation functions and a pair-weighted RMS peculiar velocity of 900 km s^{-1}. In addition, the predicted fluctuations in the microwave background are well within the allowed limits (Bond & Efstathiou 1984; Vittorio & Silk 1984).

The model sketched above is clearly rather crude and only in rough agreement with the data; it does, however, illustrate that the kind of bias imposed by a global density threshold for galaxy formation goes in the right direction and has about the right size to reconcile a flat cold dark-matter universe with observations of the galaxy distribution on megaparsec scales. This match fixes the values of h, s and ν and there are no further free parameters in the model; naturally the next question to address is whether it can also account for observations on other scales. In particular, the model ought to predict the observed number density and correlation properties of Abell clusters and the existence of large voids (see §2). The present simulations do appear to produce the right abundance of Abell clusters but they are on too small a scale to give a definitive answer. About three rich clusters are expected in a volume the size of the calculation shown in figure 7*b*. Assuming a mean luminosity density $\mathscr{L} = 0.015h^3L^*$ Mpc^{-3}, where $L^* = (1.6)\cdot 10^{10}h^{-2}L_\odot$ (Felten 1985), the three largest clumps in this figure have luminosities $L \approx 130L^*$ and velocity dispersions $v \approx 550$ km s^{-1} within radius $r = 3$ Mpc; these numbers can be compared with typical values for Abell clusters of $L \approx 150L^*$ (Sarazin 1985), $v \approx 800$ km s^{-1} (Noonan 1973) and $r = 3$ Mpc. Preliminary results from a new set of larger simulations seem to confirm that the correct abundance of clusters is produced and that the properties of voids may also be consistent with the data; the clustering amplitude of rich clusters, however, appears to be smaller than that reported for Abell clusters (see §2).

(*b*) *Galactic halos*

If it is to be viable the cold dark matter model should also be able to account for the abundance and characteristic properties of galaxy halos. This problem is well suited for study with high-resolution *N*-body techniques. The initial conditions are completely specified within the biased galaxy formation model discussed above; this gives the amplitude of the initial fluctuation spectrum and its scaling, $\Omega = 1$, $H_0 = 50$ km s^{-1} Mpc^{-1}. I will briefly describe the results of a recent simulation that shows how galactic size structures form (Frenk *et al.* 1985). We simulated a cubic region of present size 14 Mpc from a redshift of six to the present day. In the mean such a cube has a mass of $2.10^{14}M_\odot$, which we represented by 32768 particles of $7.10^9M_\odot$ each. The length resolution, determined by the force softening, was 2 kpc at the start of the calculation and 14 kpc at the end. Thus we were able to resolve scales comparable with the luminous regions of galaxies. In the real Universe, two galaxies brighter than M31 and 10 galaxies brighter than M33 are expected on average in a region the size of our calculation.

Figure 8*a* shows projections of the simulation at redshifts of 2.5, 1.0 and 0. By the first time shown, about 20 clumps with circular speeds exceeding 100 km s^{-1} have collapsed; of these,

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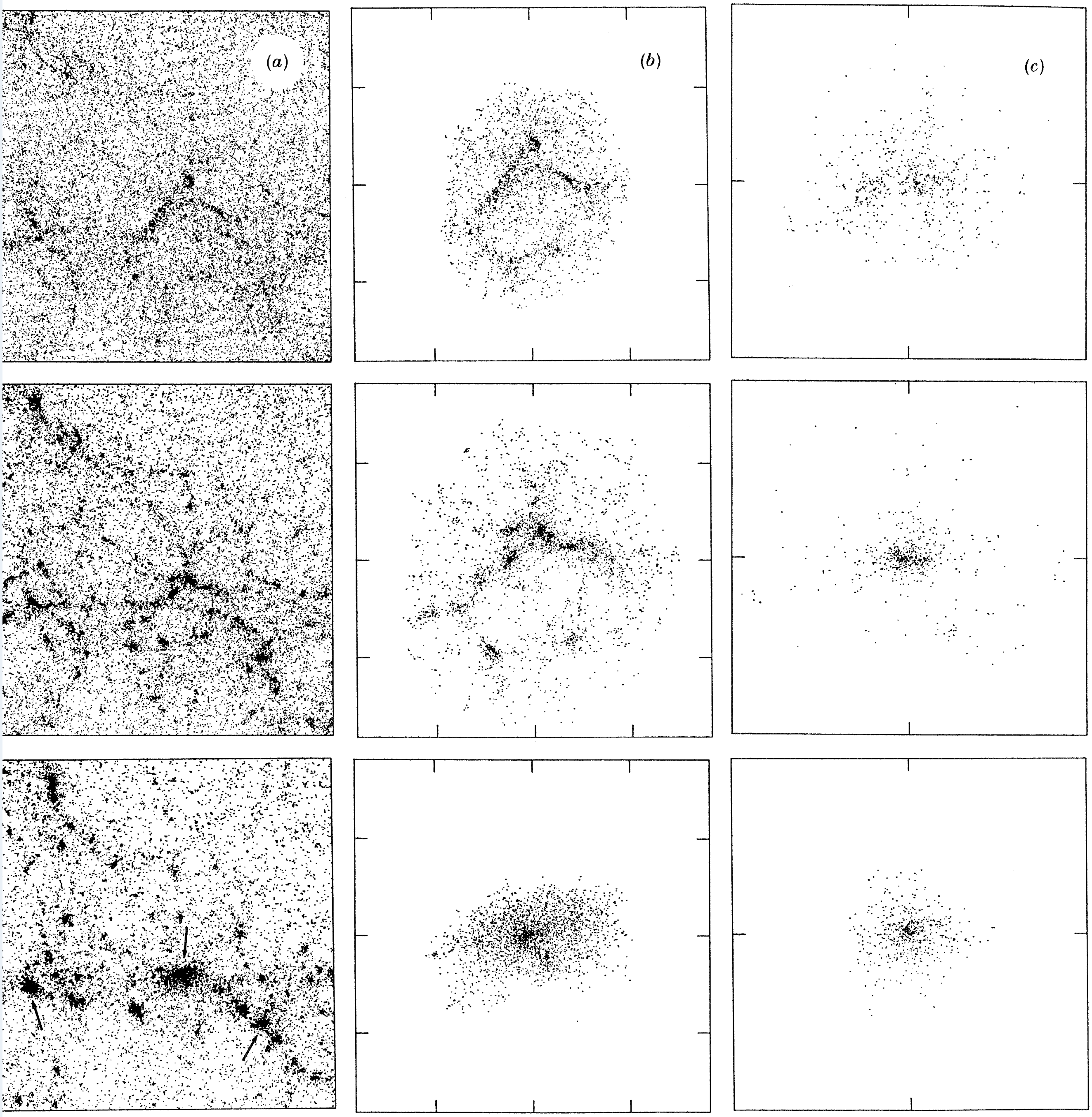

FIGURE 8. The formation of galactic halos in a flat cold dark-matter universe. (*a*) Projections in comoving coordinates of particles in a (14 Mpc)^3 volume at redshifts of 2.5, 1.0, 0. (*b*) and (*c*) positions in physical coordinates and at these same redshifts of the particles that end up in the two clumps marked with arrows at the bottom right of (*a*); tick marks represent 1 Mpc intervals. These clumps were identified at the final time from the group-finding algorithm described by Davis *et al.* (1985); their density contrast is about 500. (*b*) contains 4156 particles and is the most massive clump in the simulation; it forms by a complicated sequence of merger events and may be representative of halos around elliptical galaxies. (*c*) contains 692 particles; it evolves in relative isolation and may be representative of halos around spiral galaxies.

two have circular speeds larger than 200 km s^{-1}. These clumps are all associated with high peaks of the linear density field. About half of them remain relatively isolated accreting surrounding material as the universe evolves; the rest participate in one or more violent merging events. These two generic types of evolution are illustrated in figure 8*b*, *c*, which shows the history of all the particles that end up in the clumps marked with an arrow at the right of figure 8*a* at the final time. The present day configurations are smooth, centrally concentrated and significantly triaxial. An important feature of merger events is that a significant transfer of angular momentum from the orbits of subclumps to outlying material occurs as substructure is erased. For example, the most bound 20% of the clump in figure 8*b* lost 70% of its angular momentum between a redshift of one and the present. As a result, the inner parts of merger remnants tend to be slowly rotating.

The internal structure of galactic halos can be inferred from measurements of rotation curves in the outer parts of disk galaxies (Rubin *et al.* 1985; van Albada *et al.* 1985; see also van Albada 1986, this symposium). In figure 9 I have plotted the circular velocity, $v_c = (GM/r)^{\frac{1}{2}}$, as a function of radius for the 10 largest clumps in the simulation. Apart from the top two cases, which I will discuss below, these curves are remarkably flat at large radii and resemble the measured rotation curves of spiral galaxies. They correspond to relatively isolated clumps which could have accreted discs by infall of high angular momentum gas during extended periods of quiescent evolution (Gunn 1982); we may identify them with the massive halos of spiral galaxies. The top rotation curve in the figure corresponds to the largest clump in the simulation (figure 8*b*). This structure formed by several violent mergers and is unlikely to have ever been able to form or retain a disc; we may identify it instead with the massive halo of an elliptical galaxy. Finally, the second largest rotation curve in figure 9 corresponds to the clump marked with an arrow at the bottom left of figure 8*a*. This is in fact a binary which

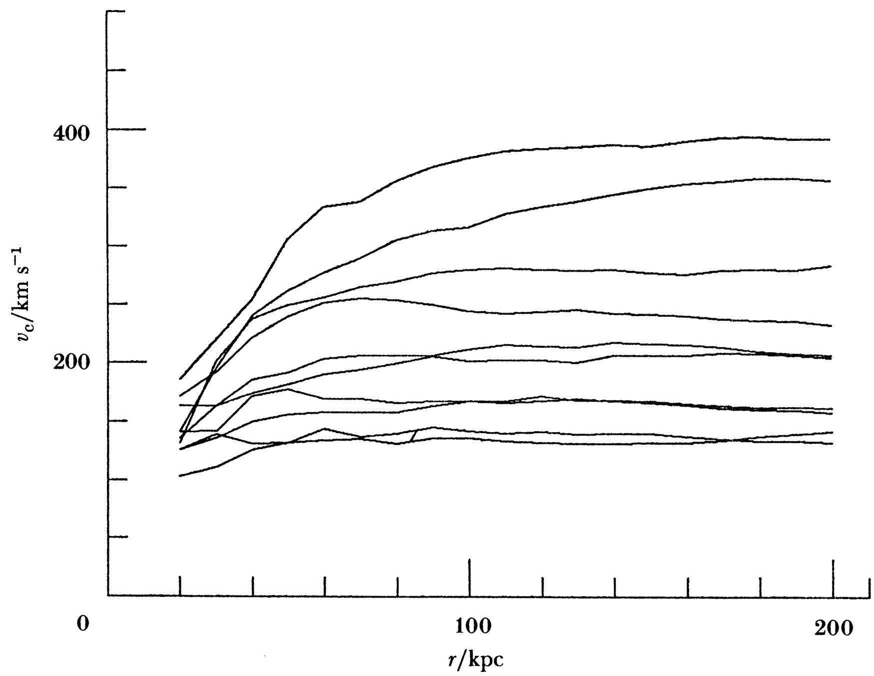

FIGURE 9. Rotation curves for the ten most massive clumps in figure 8 at the final time. Circular velocities are $v_c = [GM(r)/r]^{\frac{1}{2}}$, where $M(r)$ is the mass contained within radius r.

had nearly but not completely merged at the end of the simulation; its two components have flat rotation curves with an amplitude of 220 km s^{-1}. Of the halos in the figure, the smallest has a circular velocity similar to that of M33 whereas the third largest has a circular velocity similar to that of M31. Thus the simulation produces the observed abundance of massive halos and they have flat rotation curves with amplitudes that span the observed range. This is quite remarkable, because the free parameters of the model had been fixed beforehand by observations of galaxy clustering.

7. Conclusions

The study of galaxy clustering has elicited much interest in recent years, largely as a result of an unprecedented interaction between observations and theory. On the observational side large samples of galaxies and clusters with measured redshifts have enabled analysis of their spatial distribution up to scales of several tens of megaparsecs. On the theoretical side the standard hot-Big-Bang cosmology has been complemented by new ideas from particle physics concerning the very early Universe and the possible identity of the missing mass. Although there are, as yet, few definitive answers, new avenues have been opened. Observationally the clustering properties of optically bright galaxies on scales less than about $10h^{-1}$ Mpc are fairly well understood; the main uncertainty is whether these properties are common to most galaxies or whether, as suggested by recent studies, the characteristic clustering length varies with galaxy type. The small-scale correlations of galaxies' peculiar velocities imply a value of $\Omega \approx 0.2$ for the contribution to the mean cosmic density of matter statistically clustered as bright galaxies on scales less than about $3h^{-1}$ Mpc; this estimate, however, does not constrain a possible contribution due to a more smoothly distributed component. Rich galaxy clusters appear to be correlated even at separations of *ca.* $50h^{-1}$ Mpc, but there is, as yet, no more than anecdotal evidence for the existence of filaments and voids. Much effort is likely to be expended in future on systematic studies of these very large scales as well as in attempts to answer the important question of which, if any, luminous material fairly samples the distribution of invisible mass.

Theoretical input to studies of galaxy clustering has come partly from the inflationary model of the early Universe and partly from detailed evolutionary models of universes in which the dark matter consists of weakly interacting elementary particles. The former makes a prediction for the primordial fluctuation spectrum and has reinforced the old prejudice that the Universe ought to be flat. The latter lead to specific predictions for the galaxy distribution that can be tested against observations. Linear analyses of the growth of density perturbations in the expanding universe have been complemented by N-body simulations which can accurately follow the late nonlinear phases. I have contrasted here the formation of structure in universes dominated by various types of elementary particles with that in scale-free models. In the latter case the evolution is self-similar, but the resulting correlations differ both from earlier analytic results and, unless the primordial power spectral index $n \approx -2$, from observations of galaxy clustering. N-body simulations and other related studies have shown that light stable neutrinos with a mass of about 30 eV are unlikely candidates for the missing mass. An open universe dominated by cold dark matter in which the bright galaxies trace the mass is compatible with observations of galaxy clustering on small scales but appears to be inconsistent with the observed isotropy of the microwave background. This disagreement can be removed while preserving the agreement with the observations of galaxy clustering as well as with the

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small-scale dynamical determinations of Ω by assuming that the cold dark-matter universe is flat, but the galaxies form only near high peaks of the density field and as a result are more strongly clustered than the underlying mass distribution. This model has attracted considerable attention in recent years. In particular a high-resolution N-body simulation has shown that it produces the observed abundance of galactic halos with flat rotation curves whose amplitudes are similar to those inferred for the massive halos of spiral galaxies. The appearance of a cold dark-matter universe on scales greater than about $10h^{-1}$ Mpc has not yet been worked out in detail; voids and rich galaxy clusters appear to be formed with the observed abundance, but it is not yet clear whether the model can reproduce the reported correlation properties of Abell clusters. Other aspects of the cold dark-matter model which are amenable to direct observational testing are the possibility that voids may contain 'failed' galaxies in the form of Lα clouds or low-surface-brightness objects and the fairly recent but protracted process by which galaxies are expected to form.

Despite the many areas that remain unexplored, the cold dark-matter model with a biased galaxy formation process has been singularly successful and, if nothing else, it will have helped to identify the sort of conditions required to explain certain features of the galaxy distribution. It seems nevertheless remarkable that a scheme that pieces together physical phenomena spanning the entire lifetime of the Universe can lead to such specific predictions and that so many of them seem to conform so well to what is observed. Ultimately, the question of what is the missing mass can only be conclusively resolved by direct detection. Ellis (1986, this symposium) and Jelley (1986, this symposium) discuss some very exciting prospects for detecting cold dark-matter candidates. If such efforts eventually prove successful, they will represent a major, but perhaps not entirely unexpected, discovery.

Much of the material in §§4–6 is joint work carried out with G. Efstathiou, M. Davis and S. White over the last four years. I would like to thank them for our enjoyable collaboration and for permission to present some of our new results prior to joint publication. I would also like to thank N. Kaiser and M. J. Rees, F.R.S., for stimulating discussions. I acknowledge the warm hospitality of the Royal Greenwich Observatory, where many of the computations were performed, and of the Institute of Astronomy, Cambridge, during the summer of 1985. I acknowledge support from the SERC and from NATO grant no. 689/84 for international collaboration in research.

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